Modelling an extra dimension with domain-wall branes

Damien George

Nikhef theory seminar — 5th November 2009

Overview

Physics beyond the standard model: extra dimensions.

- Brane $\leftrightarrow$ domain wall (topological defect).
- Gravity & SM particles trapped to brane.
- At low energies, 5D $\longrightarrow$ 4D.

We will cover:

- Randall-Sundrum 2 model with delta-function brane.
- Domain walls (kinks/solitons).
- Trapping scalar and fermion fields to a domain wall.
- Using Dvali-Shifman mechanism to trap gauge fields.
- $\mathrm{SU}(5)$ grand unified domain-wall model.
- Extending $\mathrm{SU}(5)$ to E_6.
- Cosmology — the expansion of a brane universe.

Collaborators: Ray Volkas (Melbourne), Rhys Davies (Oxford), Mark Trodden (U Penn), Kamesh Wali (Syracuse, NY), Aharon Davidson (Ben-Gurion, Israel), Archil Kobakhidze, Gareth Dando (Melbourne).

Motivation

We are inspired by the Randall-Sundrum warped metric solution.

RS1 is a compact extra dimension: provides a solution to the hierarchy problem — lots of work on this model. Branes are string theory like objects. Warped throats, inflation, dark matter, ...

(Randall & Sundrum, PRL83, 3370 (1999))

RS2 is an infinite extra dimension: solves the trapping of low-energy gravity. Not as much interest because it doesn't solve any major problems, just introduces another dimension.

(Randall & Sundrum, PRL83, 4690 (1999))

We will pursue RS2 because it seems a natural extension of 3+1 space. Extra dimensions → extra degrees of freedom to solve problems: weak hierarchy, GUT (proton decay, mass relations), mass hierarchy.

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Premise:

- take the standard model and general relativity
- add an *infinite* extra space dimension
- recover the standard model and general relativity at low energies

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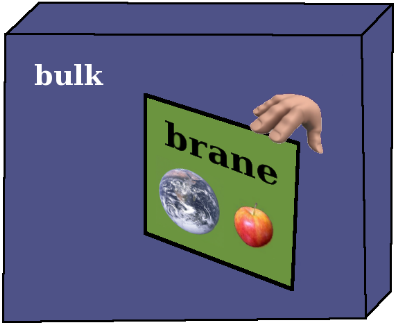

$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \left[-M^3 R + \delta(y) \mathcal{L}_{\mathsf{SM}} \right]$$

RS2 model

Need brane and bulk sources:

$$\mathcal{S} = \int d^4x \int dy \left[\sqrt{|g|}(-M^3 R - \Lambda_{\text{bulk}}) + \sqrt{|g^{(4)}|}\delta(y)(\mathcal{L}_{\text{SM}} - \Lambda_{\text{brane}}) \right] \tag{1}$$

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Solve the theory:

- Randall-Sundrum metric ansatz: $ds^2 = e^{-2k|y|} g^{(4)}_{\mu\nu} dx^\mu dx^\nu - dy^2$
- Solve Einstein's equations ($\mathcal{L}_{\text{SM}} = 0$ and $R^{(4)} = 0$):

$$\Lambda_{\text{bulk}} = -12k^2 M^3 \qquad\qquad \Lambda_{\text{brane}} = 12kM^3$$

- Write R in terms of $R^{(4)}$: $R = e^{2k|y|} R^{(4)} - 16k\delta(y) + 20k^2$

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Substitute into (1) and integrate over y:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \left[-\frac{M^3}{k} R^{(4)} + \mathcal{L}_{\text{SM}} \right]$$

A dimensionally reduced theory.

Newton's law

Just need to check Newton's law. Linear tensor fluctuations are:

$$g_{\mu\nu} = e^{-2k|y|}\eta_{\mu\nu} + \sum_n h^{(4)}_{n\ \mu\nu}(x^\mu)\psi_n(y)$$

The zero mode $h^{(4)}_{0\ \mu\nu}$ dominates the Kaluza-Klein tower.

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Newton's law is modified to:

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \frac{\epsilon^2}{r^2}\right) \qquad \text{(where } \epsilon = 1/k\text{)}$$

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$$\epsilon < 12\mu\text{m} \quad \Longrightarrow \quad k > 16 \times 10^{-3}\text{eV}$$

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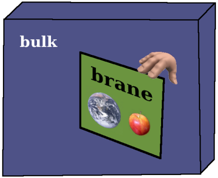
- We have what we wanted: a 5D theory that at low energies looks like our 4D universe.
- But almost no new phenomenology.
- Next step: brane forms naturally.



Thick and smooth RS2

We want to remove the $\delta(y)$ part of the action:

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Geometry, hence gravity, is 5D. So why not try to make all fields 5D?

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- Then we show how to trap scalars, fermions and gauge fields to the brane.
- Finally we present a 4+1-d $\mathrm{SU}(5)$ based extension to the standard model.

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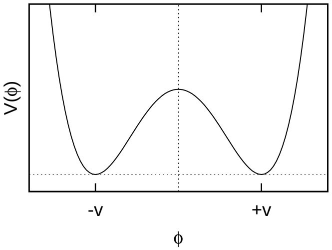
Turn off warped gravity for now — just think about trapping 5D fields.

A domain wall as a brane



A *domain-wall* is a (thin) region separating two vacua.

The field ϕ interpolates between two vacua as one moves along the extra-dimension.

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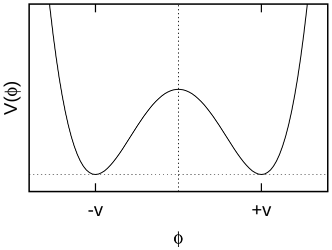

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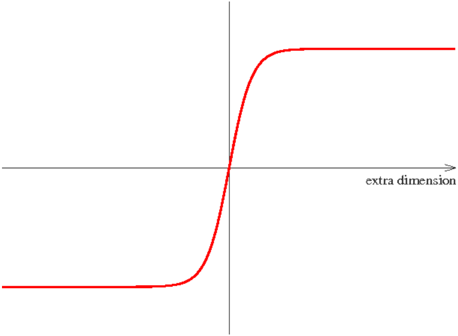
Lagrangian for $\phi(x^\mu, y)$:

$$\mathcal{L} = \frac{1}{2}\partial_M \phi \, \partial^M \phi - V(\phi)$$

A solution is the *kink*:

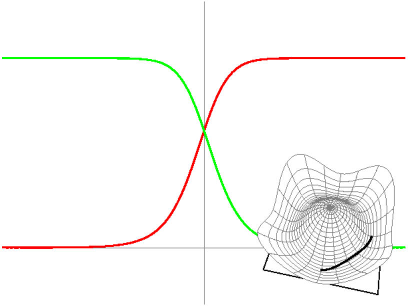
$$\phi(y) = v \tanh(\sqrt{2\lambda} v y)$$

It is stable!

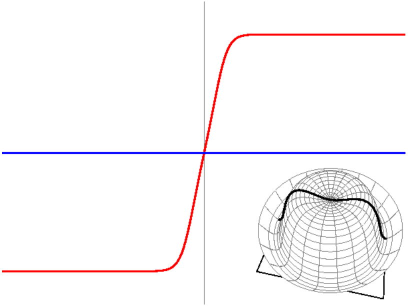


More complicated domain walls

These examples use two scalar fields to form the wall.



$$V = \lambda_1(\phi_1^2 + \phi_2^2 - v^2)^2 + \lambda_2 \phi_1^2 \phi_2^2$$



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(DPG & Volkas, PRD72, 105011 (2005))

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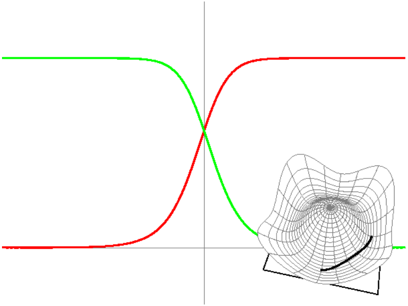

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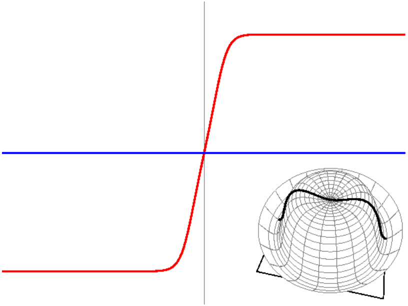

$V = \lambda(\phi_1^2+\phi_2^2-v_1^2)^2(\phi_1^2+\phi_2^2+v_2^2)$

In a realistic model, symmetries dictate V.

To determine stability, expand ϕ in normal modes about the background: $\phi(x,t) = \phi_{\mathsf{bg}}(x) + \sum_n \xi_n(x)e^{i\omega_n t}$. Make sure $\omega_n^2 \geq 0$

Trapping matter fields

Trapping scalar fields

(DPG & Volkas, PRD75, 105007 (2007))

Aim: to trap a 5D scalar field $\Xi(x^\mu, y)$ to the brane.

A simple quartic coupling works:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{1}{2}\partial^M\phi\, \partial_M\phi - V(\phi) + \frac{1}{2}\partial^M\Xi\, \partial_M\Xi - W(\Xi) - g\phi^2\Xi^2 \right]$$

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Expand Ξ in extra dimensional (Kaluza-Klein) modes:

$$\Xi(x^\mu, y) = \sum_n \xi_n(x^\mu) k_n(y)$$

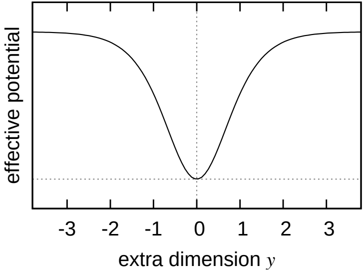
ξ_n are the 4D fields, k_n their extra-dimensional profile. The profiles satisfy a Schrödinger equation:

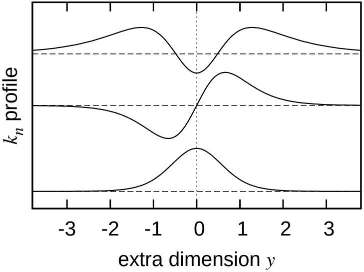
$$\left(-\frac{d^2}{dy^2} + 2g\phi^2_{\mathsf{bg}} \right) k_n(y) = E_n^2 k_n(y)$$

The energy eigenvalues E_n are related to the mass of the 4D field ξ_n.

The effective potential acts like a well.

$$\left(-\frac{d^2}{dy^2} + 2g\phi_{\mathsf{bg}}^2\right) k_n = E_n^2 k_n$$





Trapping via a potential well

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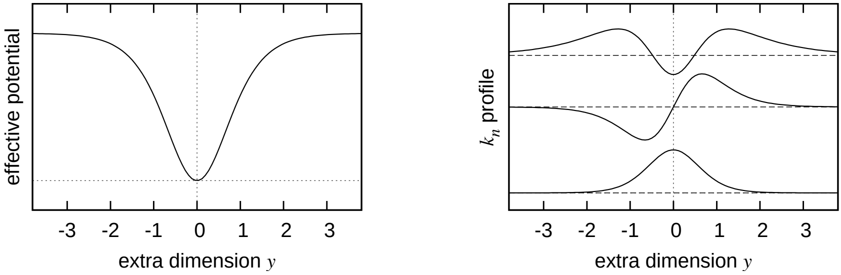

To get 4D theory, substitute mode expansion into action and integrate y:

$$\mathcal{S} = \int d^4x \left[\sum_n \left(\frac{1}{2}\partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 \right) + \text{(higher order terms)} \right]$$

- Orthonormal basis $k_n \implies$ diagonal kinetic and mass terms.
- m_n can be tuned.

Trapping fermions

(DPG & Volkas, PRD75, 105007 (2007))

We can trap a fermion $\Psi(x^\mu, y)$ to the brane with a Yukawa coupling:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{1}{2}\partial^M\phi \, \partial_M\phi - V(\phi) + \overline{\Psi} i \Gamma^M \partial_M \Psi - h\phi\overline{\Psi}\Psi \right]$$

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Decompose into left- and right-chiral fields and Kaluza-Klein modes:

$$\Psi(x^\mu, y) = \sum_n \left[\psi_{Ln}(x^\mu) f_{Ln}(y) + \psi_{Rn}(x^\mu) f_{Rn}(y) \right]$$

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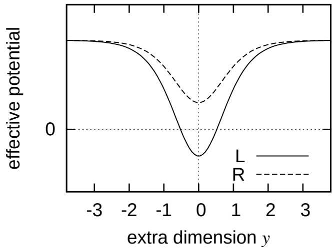
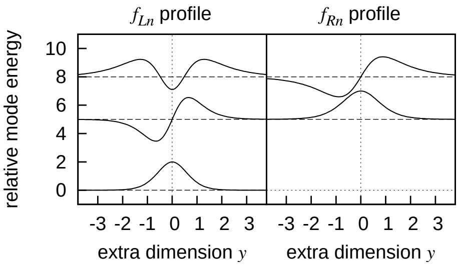
Schrödinger equation (mode index n suppressed):

$$\left(-\frac{d^2}{dy^2} + (h^2\phi_{\text{bg}}^2 \mp h\phi'_{\text{bg}}) \right) f_{L,R}(y) = m^2 f_{L,R}(y)$$





Gravity and matter fields

Brane (domain wall/kink), trapped scalar and fermion. Plus gravity:

$$\mathcal{S} = \int d^4x \int dy \sqrt{|g|} \Big[- M^3 R - \Lambda_{\mathsf{bulk}} + \frac{1}{2}\partial^M \phi \, \partial_M \phi - V(\phi) + \frac{1}{2}\partial^M \Xi \, \partial_M \Xi - W(\Xi) - g\phi^2 \Xi^2 + \overline{\Psi} i \Gamma^M \partial_M \Psi - h\phi \overline{\Psi}\Psi \Big]$$

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Dimensionally reduce by integrating over y:

$$\mathcal{S} = \int d^4x \sqrt{|g^{(4)}|} \Big[- M_{\text{4D}}^2 R^{(4)} + (\text{brane dynamics}) + \frac{1}{2}\partial^\mu \xi_n \partial_\mu \xi_n - m_n^2 \xi_n^2 - \tau_{mnop}\xi_m\xi_n\xi_o\xi_p - (\text{brane interactions}) + \overline{\psi}_{L0} i\gamma^\mu \partial_\mu \psi_{L0} + \overline{\psi}_n (i\gamma^\mu\partial_\mu - \mu_n)\psi_n - (\text{brane interactions}) \Big]$$

4D parameters (M_{4D}, m_n, τ_{mnop}, μ_n, brane dynamics) determined by eigenvalue spectra and overlap integrals.

Warped matter

Warped metric $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$ modifies profile equation:

$$\left(-\frac{d^2}{dy^2} + 5\sigma'\frac{d}{dy} + 2\sigma'' - 6\sigma'^2 + U(y)\right) f_{Ln}(y) = m_n^2 e^{2\sigma} f_{Ln}(y)$$

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(Davies & DPG, PRD76, 104010 (2007))

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Conformal coordinates $ds^2 = e^{-2\sigma(y(z))}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$.
Rescale $f_{Ln}(y) = e^{2\sigma}\tilde{f}_{Ln}(z)$:

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Matter trapping potentials are warped down.

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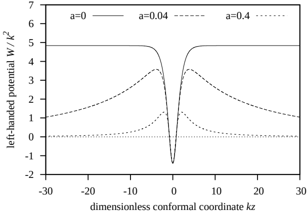
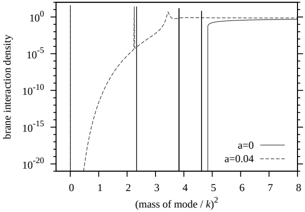
Matter trapping potentials are warped down.

- Finite bound state lifetimes.
- Resonances.
- Tiny probability of interaction with continuum.





($a \sim 1/M^3 \sim$ 5D Newton's constant)

Trapping gauge fields

Confining gauge fields

Need to trap gauge fields or e.g. Coulomb potential would be $V_{\text{Coulomb}} \sim 1/r^2$.

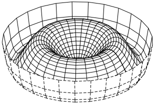
Not as simple as a Kaluza-Klein mode expansion:

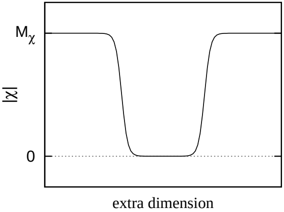
- Photon and gluons *must* remain massless.
- Need to preserve gauge universality at 3+1-d level.

We use the Dvali-Shifman mechanism, Dvali & Shifman, PLB396, 64 (1997).

The following discussion is based on Dvali, Nielsen & Tetradis, PRD77, 085005 (2008).

$\mathrm{U}(1)$ gauge theory (think 5d photon).
Charged Higgs χ (gives mass to photon).





Abelian Higgs model

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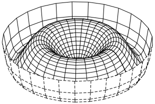
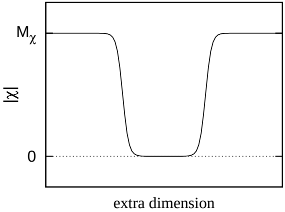
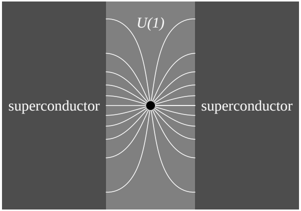






In the bulk:

- $\mathrm{U}(1)$ is broken, massive photon $\sim M_\chi$.
- Higgs vacuum is a superconductor.
- Electric charges are screened.

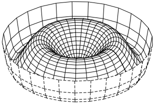
On the brane:

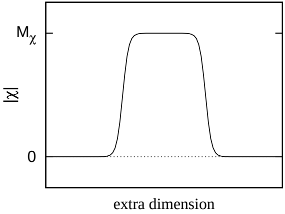
- $\mathrm{U}(1)$ is restored, massless photon.
- Electric field ends on Higgs vacuum.

Charge screening leaks onto the brane!

Using a dual superconductor

$\mathrm{SU}(2)$ gauge theory (3 "gluons").
Adjoint Higgs χ^a $(a = 1, 2, 3)$ (gives mass).





$\mathrm{SU}(2)$ gauge theory (3 "gluons").
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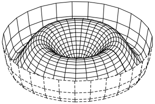
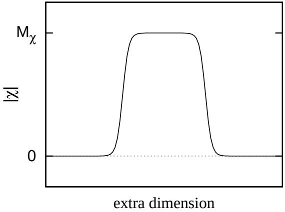
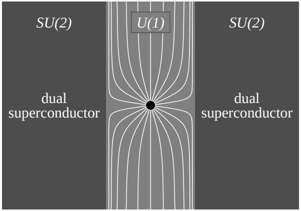






In the bulk:

- $\mathrm{SU}(2)$ is restored, in confining regime.
- Large mass gap $\sim M_\chi$ to colourless state.
- QCD-like vacuum is dual superconductor.

On the brane:

- $\mathrm{SU}(2)$ broken to $\mathrm{U}(1)$, massless photon.
- Electric field repelled from dual superconductor.

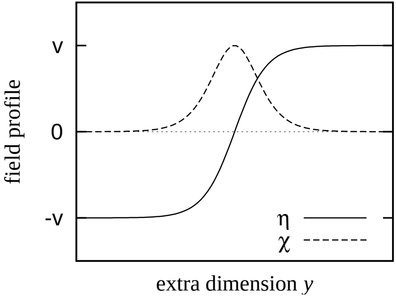
For distances much larger than brane width, electric potential $\sim 1/r$.

Dvali-Shifman model

Stabilise the domain wall with an extra uncharged scalar field η:

$$\mathcal{S} = \int d^4x \int dy \left[\frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \frac{1}{2}(D^M \chi^a)^\dagger D_M \chi^a - \lambda(\eta^2 - v^2)^2 - \frac{\lambda'}{2}(\chi^a \chi^a + \kappa^2 - v^2 + \eta^2)^2 \right]$$

- η has a kink profile.
- If $\kappa^2 - v^2 < 0$, χ becomes tachyonic near domain wall (where $\eta \sim 0$).
- True vacuum has $\chi \neq 0$ near domain wall.
- χ breaks symmetry near wall and confines gauge fields.



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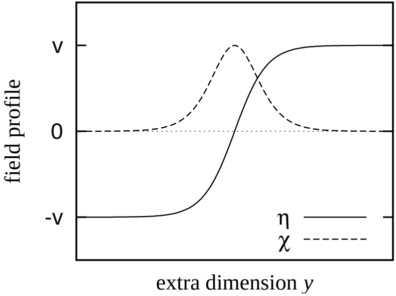

Can add gravity: self consistently solve σ (warped metric profile), η, χ.

Dvali-Shifman mechanism

The Dvali-Shifman mechanism:

- Works with any non-Abelian $\mathrm{SU}(N)$ theory.
- Assumes the $\mathrm{SU}(N)$ theory is confining (not proven for 5D).
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Obvious choice for $\mathrm{SU}(N)$ group is $\mathrm{SU}(5)$.

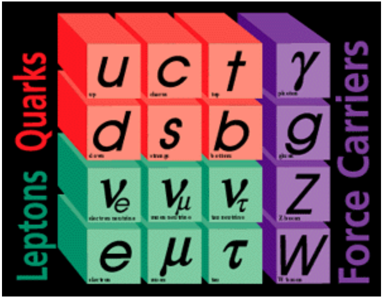
$\mathrm{SU}(5)$ basics

Quantum numbers of the standard model

Representations under $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$:

$$q_L \sim (\mathbf{3}, \mathbf{2})_{1/3} \quad u_R \sim (\mathbf{3}, \mathbf{1})_{4/3} \quad d_R \sim (\mathbf{3}, \mathbf{1})_{-2/3}$$

$$l_L \sim (\mathbf{1}, \mathbf{2})_{-1} \quad \nu_R \sim (\mathbf{1}, \mathbf{1})_0 \quad e_R \sim (\mathbf{1}, \mathbf{1})_{-2}$$



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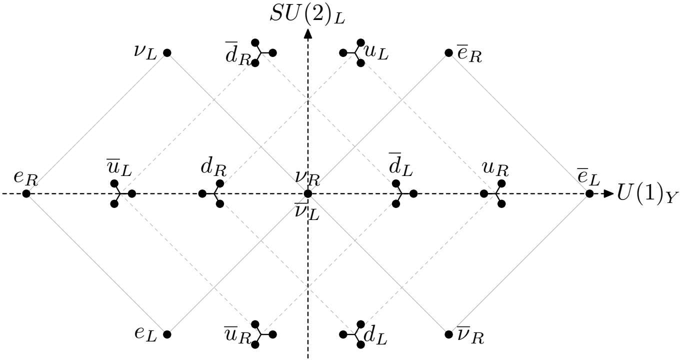

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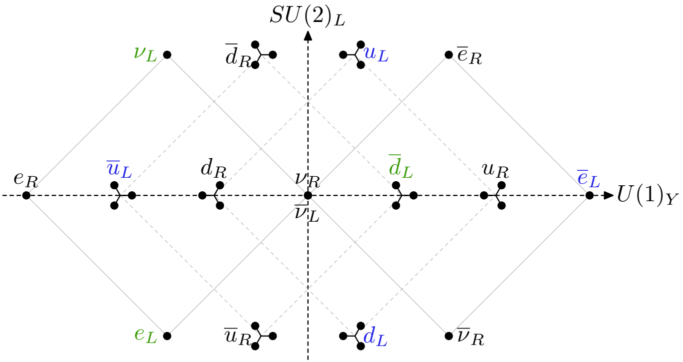

$$\mathbf{5}^* \supset \overline{d}_L^{r,w,b} \quad \nu_L \quad e_L$$

$$(\mathbf{5}\times\mathbf{5})_A = \mathbf{10} \supset \overline{u}_L^{r,w,b} \quad u_L^{r,w,b} \quad d_L^{r,w,b} \quad \overline{e}_L$$

Putting it all together

The $\mathrm{SU}(5)$ model

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Want the standard model on the brane: $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

Dvali-Shifman needs a *larger* gauge group in the bulk:

$\mathrm{SU}(5)$ is a perfect fit!

Unify the fermions as usual: $\mathbf{5}^*$, $\mathbf{10}$.
Higgs doublet goes in a $\mathbf{5}^*$.

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Summary:

- $4+1$-dimensional theory — all spatial dimensions the same.
- $\mathrm{SU}(5)$ local gauge symmetry, $\mathbb{Z}_2$ discrete symmetry.
- Field content:
  - gauge fields: $G_{MN} \sim \mathbf{24}$.
  - scalars: $\eta \sim \mathbf{1}$, $\chi \sim \mathbf{24}$, $\Phi \sim \mathbf{5}^*$.
  - fermions: $\Psi_5 \sim \mathbf{5}^*$, $\Psi_{10} \sim \mathbf{10}$.
- The standard model emerges as a low energy approximation.

Ignore gravity for now.

The action (without gravity)

(Davies, DPG & Volkas, PRD77, 124038 (2008))

The theory is described by:

$$\begin{aligned}
\mathcal{S} = \int d^4x \int dy \Bigg[& \frac{-1}{4g^2} G^{aMN} G^a_{MN} + \frac{1}{2}\partial^M \eta \partial_M \eta + \mathrm{Tr}\left((D^M \chi)^\dagger (D_M \chi) \right) \\
& + (D^M \Phi)^\dagger (D_M \Phi) + \overline{\Psi}_5 i \Gamma^M D_M \Psi_5 + \overline{\Psi}_{10} i \Gamma^M D_M \Psi_{10} \\
& - h_{5\eta} \overline{\Psi}_5 \Psi_5 \eta - h_{5\chi} \overline{\Psi}_5 \chi^T \Psi_5 \\
& \qquad - h_{10\eta} \,\mathrm{Tr}(\overline{\Psi}_{10} \Psi_{10}) \eta + 2 h_{10\chi} \,\mathrm{Tr}(\overline{\Psi}_{10} \chi \Psi_{10}) \\
& - h_- \overline{(\Psi_5)^c} \Psi_{10} \Phi - h_+ (\epsilon \overline{(\Psi_{10})^c} \Psi_{10} \Phi^*) + \text{h.c.} \\
& - (c\eta^2 - \mu_\chi^2) \,\mathrm{Tr}(\chi^2) - d\eta \,\mathrm{Tr}(\chi^3) \\
& \qquad - \lambda_1 \left[\mathrm{Tr}(\chi^2) \right]^2 - \lambda_2 \,\mathrm{Tr}(\chi^4) - l(\eta^2 - v^2)^2 \\
& - \mu_\Phi^2 \Phi^\dagger \Phi - \lambda_3 (\Phi^\dagger \Phi)^2 - \lambda_4 \Phi^\dagger \Phi \eta^2 \\
& \qquad - 2\lambda_5 \Phi^\dagger \Phi \,\mathrm{Tr}(\chi^2) - \lambda_6 \Phi^\dagger (\chi^T)^2 \Phi - \lambda_7 \Phi^\dagger \chi^T \Phi \eta \Bigg]
\end{aligned}$$

with kinetic, brane trapping, mass and Dvali-Shifman terms.

Split fermions

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Let Ψ_{nY} be the components of Ψ_5 and Ψ_{10} ($n = 5, 10$, Y = hypercharge of component), e.g. $\Psi_5 \supset \Psi_{5,-1} = l_L$. Dirac equation:

$$\left[i\Gamma^M \partial_M - h_{n\eta}\eta(y) - \sqrt{\frac{3}{5}}\frac{Y}{2}\, h_{n\chi}\chi_1(y) \right] \Psi_{nY}(x^\mu, y) = 0$$

Each Ψ_{nY} is a non-chiral 5D field: need to extract the confined left-chiral zero-mode (recall the mode expansion and Schrödinger equation approach):

$$\Psi_{nY}(x^\mu, y) = \psi_{nY,L}(x^\mu) f_{nY}(y) + \text{massive modes}$$

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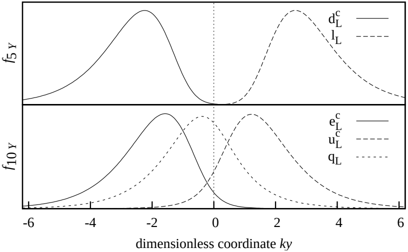
The effective Schrödinger potential depends on Y.

Thus each component $\psi_{nY,L}$ has a different profile f_{nY}.



Φ contains the Higgs doublet Φ_w and a coloured triplet Φ_c. Mode expand $\Phi_{w,c}(x^\mu, y) = \phi_{w,c}(x^\mu) p_{w,c}(y)$. Schrödinger equation for $p_{w,c}$ is:

$$\left(-\frac{d^2}{dy^2} + \frac{3Y^2}{20}\lambda_6 \chi_1^2 + \sqrt{\frac{3}{5}}\frac{Y}{2}\lambda_7 \eta \chi_1 + \dots \right) p_{w,c}(y) = m_{w,c}^2 p_{w,c}(y)$$

Critical that ground states have:

- $m_w^2 < 0$ to break electroweak symmetry.
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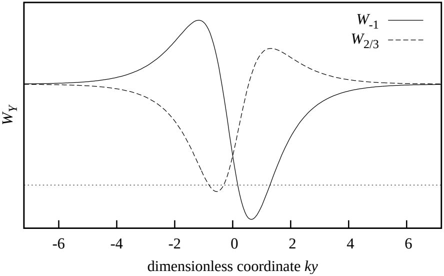
Large enough parameter space to allow this.



Features

(Davies, DPG & Volkas, PRD77, 124038 (2008))

Standard model parameters are computed from overlap integrals.

With one generation of fermions, parameters are easy to fit.

The model overcomes the major $\mathrm{SU}(5)$ obstacles:

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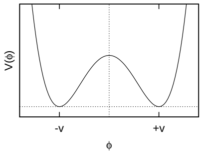
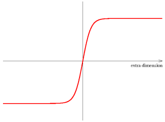
Adding gravity:

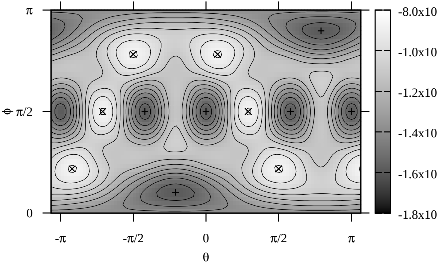
- Solve for warped metric, kink and Dvali-Shifman background.
- Continuum fermion and scalar modes are highly suppressed on the brane.
- Main features remain.

One promising extension is to the E_6 group:

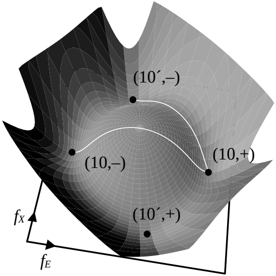
- $E_6 \rightarrow SO(10)$ in the bulk.
- $SO(10) \rightarrow SU(5)$ on the brane due to clash-of-symmetries (CoS) and Dvali-Shifman.
- Can eliminate kink scalar field η.
- Can unify Ψ_5 and Ψ_{10}.
- Large reduction of free parameters.







($+$'s are $SO(10)$'s)



(top domain-wall has CoS)

Domain-wall cosmology

The scale factor on a brane

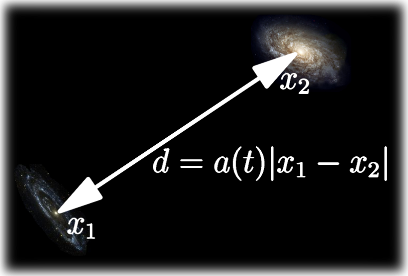
Most important cosmological fact: expanding universe.

FRW: $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x} \cdot d\mathbf{x}$

Energy density ρ dictates expansion:

$$H = \frac{\dot{a}}{a} \qquad H^2 = \frac{8\pi G}{3}\rho$$

(Hubble parameter) (spatially flat universe)



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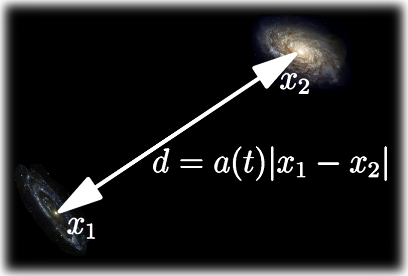


FRW for a brane: $ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\, d\mathbf{x} \cdot d\mathbf{x} + dy^2$

$$H^2_{\text{brane}} = \frac{8\pi G}{3}\left(1 + \frac{\rho}{2\sigma}\right)\rho$$

σ is the energy density (tension) of the brane; $\sigma \gg (1MeV)^4$ from BBN.

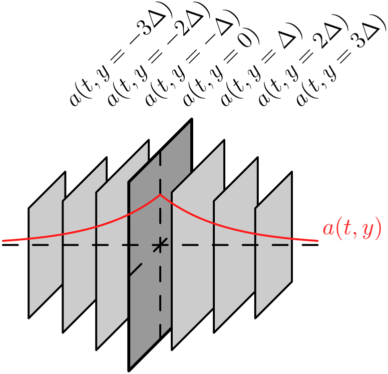
(Binétruy, Deffayet & Langlois, Nucl. Phys. B565, 269 (2000))

Species-dependent scale factor

(DPG, Trodden & Volkas, JHEP02(2009)035)

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)d\mathbf{x}\cdot d\mathbf{x} + dy^2$$

Each slice at constant y has a different scale factor.



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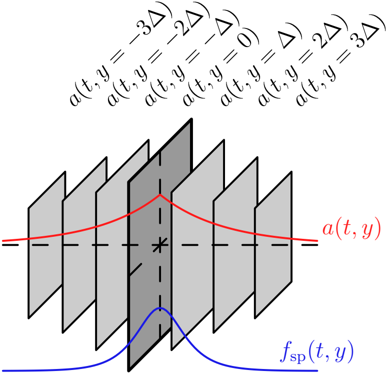

For thick branes, different species (electron, quark, KK mode) experience different expansion rates.

$$a_{\sf sp}(t) = a_0(t)\left(1 + \frac{\dot{\rho}}{2\sigma H_0} I_{\sf sp}(t)\right)$$

$$I_{\sf sp}(t) = \frac{\int f_{\sf sp}^2(t,y)\; dy}{\int f_{\sf sp}^2(t,y)\; e^{-\sigma|y|/6M_5^3} dy} - 1$$

Different localisation profile $f_{\sf sp}$ for different energies! No chiral fermions!

Summary

Our universe may be a brane residing in a higher dimensional bulk.

We have investigated the possibility that the brane is in fact a domain-wall.

Main results:

- Scalar field: forms stable domain wall.
- RS2 warped metric: traps gravity.
- Dvali-Shifman mechanism: traps gauge fields.
- $\mathrm{SU}(5)$ domain-wall model: overcomes major $\mathrm{SU}(5)$ obstacles.
- E_6 extension: unify fields and reduce parameters.
- Cosmology: species-dependent expansion rate.

Extra-dimensions a real possibility!
Will be tested by LHC (e.g. KK modes) and cosmological observations.

Future work

Open questions:

- Further analysis of the cosmology.
- Completion of E_6 model.

Incorporate SUSY: (G. R. Dvali & M. A. Shifman, *Nucl. Phys.* **B504** (1997) 127–146)

- SUSY domain wall $\rightarrow$ dimensional reduction $+$ SUSY breaking.
- SUSY broken only on domain wall, where 4D particles are localised.
- 5D superfield projected to localised 4D fermion (index theorem).
- Solution to DM and hierarchy problems within DW construction.
- Phenomenological predictions at LHC energies.